Mathematics - Paper-I A (First year)

Time: 3 Hours **Max.Marks: 75**

SECTION - A

I.Very Short Answer Questions. Answer all questions. ***10×2 = 20 M***

1. Find the ranges of real valued functions $\frac{x^2-4}{x-2}$
2. If f(x) = 2, g(x) = x^2, h(x) = 2x for all $x \in R$, then find ((fo(goh)(x)).
3. A certain book shop has 10 dozen Chemistry books, 8 dozen Physics books, 10 dozen Economics books. Their selling prices are Rs.80, Rs.60 and Rs.40 each respectively. Find the total amount of the book shop will receive by selling all the books using matrix algebra.
4. If $\begin{bmatrix} 0 & 1 & 4 \\ -1 & 0 & 7 \\ -x & -7 & 0 \end{bmatrix}$ is a skew symmetric matrix, then find x.
5. If $4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$ is parallel to the vector $\bar{i} + 2\bar{j} + 3\bar{k}$, find P.
6. Let $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$ and $\bar{b} = 3\bar{i} + \bar{j}$. Find the unit vector in the direction of $\bar{a} + \bar{b}$.
7. Let $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ and $b = 2\vec{i} + 3\vec{j} + \vec{k}$ Find the projection vector of $\vec{b}$ on $\vec{a}$ and its magnitude.
8. Find the value of $\sin^2 82^\circ \frac{1}{2} - \sin^2 22^\circ \frac{1}{2}$
9. Find the period of f(x) = $\sin^4 x + \cos^4 x$ for any $x \in R$
10. Prove that $\cosh^4 x - \sinh^4 x = \cosh(2x)$

SECTION - B

II.Short Answer Questions. Answer any 'Five' Questions. ***5×4 =20 M***

11. Show that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$
12. Prove by vector method that $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line in

intercept form.

13. Find the volume of the tetrahedron whose vertices are (1, 2, 1) (3, 2, 5), (2, –1, 0) and (–1, 0, 1)
14. If A+B = π /4 , then prove that (1+ tan A) (1+ tan B) = 2.
15. If tan (π cos x) = cot (π sin x), prove that $\cos\left(x-\frac{\pi}{4}\right)=\pm\frac{1}{2\sqrt{2}}$
16. Prove that $\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)=\sin^{-1}\left(\frac{77}{85}\right)$
17. Prove that cot A+ cotB+ cot C $=\frac{a^2+b^2+c^2}{4\Delta}$

SECTION - C

III.Long Answer Questions. Answer any 'Five' Questions. ***5 ×7 = 35 M***

18. If f:A $\to$ B and g:B $\to$ C are bijections then prove that $(gof)^{-1} = f_0^{-1}\, g^{-1}$
19. Prove by Mathematical Induction 1.3 + 3.5 + 5.7 +...up to n terms $=\frac{n\left(4n^2+6n-1\right)}{3}$
20. Solve by matrix inverse method

    5x – 6y + 4z =15

    7x + 4y – 3z =19

    2x + y + 6z = 46
21. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \left(a^3+b^3+c^3-3abc\right)^2$
22. Find the shortest distance between the skew lines $\bar{r}=\left(6\bar{i}+2\bar{j}+2\bar{k}\right)+t\left(\bar{i}-2\bar{j}+2\bar{k}\right)$ and $\bar{r}=\left(-4\bar{i}-\bar{k}\right)+s\left(3\bar{i}-2\bar{j}-2\bar{k}\right)$
23. If A+B + C = π prove that $\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}=4\cos\frac{\pi-A}{4}\cos\frac{\pi-B}{4}\cos\frac{\pi-C}{4}$
24. Show that

    a^3 cos (B–C) + b^3 cos (C –A) + c^3 cos (A–B) = 3abc.